Comparison of Transition Matrices of Land-use Dynamics in Two Research Sites

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Introduction

Several international research projects on the intensity of land use change have been started since the late 1980’s, e.g. the Land Use and Cover Change program (Messerli, 1997) of International Geosphere-Biosphere Project (1988). These studies indicated the necessity of intensive studies on land-use changes, their speed and their driving forces. Developments all over the country cause the loss of a significant amount of agriculture land and natural forest. Local land-use and land-cover change can influence environmental and ecological changes. Two research sites we selected, Lambir National Park (Malaysia) and Deramakot Forest Reserve (Malaysia), have also been affected by the influence over 50 years, and the historical, social and ecological reasons would be very different depending on the locality and periods. To identify those reasons, quantitative research is necessary on the speed and the driving forces of their changes. As the first step to examine them, we here tried employing a mathematical model approach using transition matrix models.

Transition matrix model is a useful tool to analyze the characteristics of the land-use change and to forecast future dynamics and is easily constructed from area-based transition tables among land-use classifications. Popular GIS software, e.g. ARCGIS and RAMASGIS, supports a command to calculate the area-based transition tables from satellite images and aerial photographs. Therefore, many authors employed transition matrix models in these fifteen years (Ehlers et al., 1990; Meyer 1991; Hathout, 2002; Ademola, 2004).

We, in the present paper, quantified area-based transition tables and transition matrices of land-use in two research sites in order to grasp historical change of transition probabilities among census periods. To evaluate the historical change of transition probabilities properly, they should be compared under the same census interval. Unfortunately, satellite images or photographs of the research site in question are not always prepared every year or by a constant time interval in the specific research site. It’s necessary to adjust the difference of census intervals of transition matrices mathematically. Therefore, we developed a formula of obtaining the yearly (annual) transition matrix to compare them among census periods. Using the formula, two yearly matrices in the research sites were obtained and the ten-year matrices were also calculated.

Methods

(i) Area-based transition tables and transition matrices of land use

We can quantify the type, amount and location of land use change in satellite images and aerial photographs, and some GIS computer programs provide a procedure to classify the land use and land cover, and to calculate the transition in the areas of the classifications of land use. We have three aerial photographs (of 1963, 1977 and 1997; Table 1a and 1b) in a part of Lambir National Park, Malaysia (about 275 square kilo-meters) and only two satellite images (of 1985 and 2002; Table 1c) in a part of Deramakot Forest Reserve, Malaysia (about 3500 square kilo-meters). Therefore, we calculated the area-based transition tables from those satellite images and aerial photographs of two research sites, and obtained 3 area-based transition tables in all (Table 1).

Probability-based transition tables, i.e. transition matrices, were also obtained from Table 1 (Table 2) because they are very convenient in comparing among research sites with different sizes and in calculating the dynamical projection of classifications of land use as:

$$\mathbf{x}_{t+c} = \mathbf{A}\mathbf{x}_t \quad , \qquad (1)$$

where $\mathbf{x}_t$, $\mathbf{A}$ and c represent the area vector at a census each of whose elements is the area of each classification, a transition matrix in question and the census interval, respectively. "$t+c$" in equation (1) generally means the next census. Unfortunately, the census intervals in Table 2 ranged from 14 to 20 years because of the lack of satellite images or aerial photographs. It implies that these obtained matrices cannot be compared directly because the transition probability during 14 years would be actually different from that during 20 years even if they are the same.

(ii) Formula of yearly transition matrix

The discrepancy of census periods in transition matrices does not allow us to evaluate two of them directly. Therefore, the normalization of census periods is necessary, that is, to obtain yearly transition matrix in every census period, which means the *c*-th power root of an original transition matrix, where *c* is census interval of the matrix. We developed a theorem on the *c*-th power root of a matrix and obtained the formula and the number of the solutions:

<u>Theorem</u> If a *n* by *n* matrix has *n* distinct eigenvalues and all of them are not equal to zero, the *c*-th power root of the matrix is:

$$\mathbf{A}^{\frac{1}{c}} = \begin{pmatrix} \mathbf{u_1} & \cdots & \mathbf{u}_n \end{pmatrix} \begin{pmatrix} (\lambda_1)^{1/c} & & 0 \\ & \ddots & \\ 0 & & (\lambda_n)^{1/c} \end{pmatrix} \begin{pmatrix} \mathbf{u_1} & \cdots & \mathbf{u}_n \end{pmatrix}^{-1} , \qquad (2)$$

where $\mathbf{A}$, λ_i and $\mathbf{u}_i$ are a transition matrix with census interval of *c* years, the *i*-th eigenvalue of matrix $\mathbf{A}$ and its corresponding eigenvector, respectively.

Table 1 Area-based transition tables among land use classifications.
The numerics in cells represents the area of transition from a classification to another (ha).

(a) From 1963 to 77 in Lambir National Park. The total area is about 275 square kilo-meters

	1963				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland	230.2	683.3	588.1	0	0
Secondary Forest	724.9	3,027.4	2,016.9	0	0
Natural Forest	1.7	4.2	14,226.0	0	0
Selectively logged forest	0.0	108.6	4,222.4	0	0
Plantation	0.0	130.6	1,446.0	0	0
Total	956.8	3,954.1	22,499.4	0.0	0.0

(b) From 1977 to 97 in Lambir National Park. The total area is about 275 square kilo-meters

	1977				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland	245.2	211.3	2.2	5.8	0.0
Secondary Forest	1,253.5	5,517.8	888.9	1,965.7	1,524.8
Natural Forest	0.0	6.7	6,364.5	0.0	0.0
Selectively logged forest	0.3	11.6	6,524.2	2,356.8	0.0
Plantation	0.0	13.4	556.3	0.0	51.8
Total	1,499.0	5,760.8	14,336.1	4,328.3	1,576.6

(c) From 1985 to 2002 in Deramakot Forest Reserve.
The total area is about 3500 square kilo-meters.

	1985				
	Cropland	Secondary Forest	Natural Forest	Selectively logged forest	Water
Cropland	41370.6	2259.3	49166.2	35269.8	777.5
Secondary Forest	563.2	45.8	549.7	146.5	3.3
Natural Forest	6189.9	1855.0	40801.9	19146.1	86.1
Selectively logged forest	17560.2	1558.2	50514.1	77961.3	162.4
Water	422.3	136.4	30.0	103.1	886.0
Total	66106.3	5854.7	141061.8	132626.8	1915.3

Table 2 Transition matrices among land use classifications.

The numerics in cells represents the transition prpbability from a classification to another.

(a) From 1963 to 77 in Lambir National Park.

	1963				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland	0.241	0.173	0.026	0	0
Secondary Forest	0.758	0.766	0.090	0	0
Natural Forest	0.002	0.001	0.632	0	0
Selectively logged forest	0.000	0.027	0.188	0	0
Plantation	0.000	0.033	0.064	0	0

(b) From 1977 to 97 in Lambir National Park.

	1977				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland	0.164	0.037	0.000	0.001	0.000
Secondary Forest	0.836	0.958	0.062	0.454	0.967
Natural Forest	0.000	0.001	0.444	0.000	0.000
Selectively logged forest	0.000	0.002	0.455	0.545	0.000
Plantation	0.000	0.002	0.039	0.000	0.033

(c) Deramakot Forest Reserve in Malaysia.

	1985				
	Cropland	Secondary Forest	Natural Forest	Selectively logged forest	Water
Cropland	0.626	0.386	0.349	0.266	0.406
Secondary Forest	0.009	0.008	0.004	0.001	0.002
Natural Forest	0.094	0.317	0.289	0.144	0.045
Selectively logged forest	0.266	0.266	0.358	0.588	0.085
Water	0.006	0.023	0.000	0.001	0.463

Collorary If a n by n matrix has n distinct eigenvalues and all of them are not equal to zero, the number of the c-th power root of the matrix is c^n.

<The proof is abbreviated>

$(\lambda_i)^{1/c}$ means the c-th power root of the scalar λ_i. Since λ_i could be a complex number, we may set

$\lambda_i = re^{i\theta} = r(\cos\theta + i\sin\theta)$ ($r > 0$ and $0 \le \theta < 2\pi$), using polar coordinates. Therefore, $(\lambda_i)^{1/c} = r^{1/c}(\cos\frac{\theta + 2\pi k}{c} + i\sin\frac{\theta + 2\pi k}{c})$ for $k = 0,1,\ldots c-1$ and generally has c solutions including complex numbers, which is why the number of the solutions are c^n.

We also developed computer programs of Mathematica (Wolfrum Research, Inc.) and C++ to calculate equation (2) and to obtain all of c^n solutions at the same time. It is opened on the website, http://hosho.ees.hokudai.ac.jp/~takada/eindex.html. The program by C++ is about 800 times faster than that by Mathematica.

Result

(i) Quantification of land use change

The quantification of land use change for the analyzed classifications are given in Table 1 and 2. The former is an area-based table and the latter is a probability-based table, i.e. transition matrix. “Natural forest” was the largest class in Lambir and Deramakot of Malaysia (Table 1a, 1b and 1c). The speed of land use change cannot be compared using the area-based tables because their total areas are different among two research sites, and cannot be compared using the probability-based tables because their census periods varied largely. We refer to the comparison of the speeds in land use change later, when we obtain the yearly transition matrices and ten-year transition matrices.

(ii) Yearly transition matrix and several problems

Yearly matrix has plural solutions and, for example, the yearly matrix in Lambir Park during 1977 and 1997, has 20^5 (= 3,200,000) solutions, elements of which could include negative and complex numbers, as explained in Method. The elements of a correct yearly matrix should range from 0 to 1 because those elements are probabilities. Therefore, we should omit solutions with negative or complex numbers after the calculation. The computer program is actually made to omit matrices with large negative real parts or imaginary parts, taking into account of rounding errors in numerical calculation.

At the second stage (1977-97) in Lambir National Park, we unfortunately obtained no positive solution. A solution among 20^5 solutions includes only one negative element whose absolute value is very small (Table 3b), and all of the other solutions include elements smaller than minus 0.5 and/or complex numbers. We think the former is the appropriate solution and its negative elements might be brought from rounding errors in numerical calculation or the failure of image analysis in land use classification. In Deramakot (Table 3c), we similarly have single appropriate solution among about a million solutions. In Lambir National Park (Table 3a), we couldn’t obtain the yearly transition matrix at the first stage, using equation (2), because the transition matrix during 1963-77 in Table 2a has two zero eigenvalues

Most of diagonal elements of yearly transition matrices are larger than 90% and the land-use changes in all the research sites is very slow by yearly rate. We calculated the ten-year matrices (10-th power matrices of yearly ones) in all the sites to understand the speed of land-use change intuitively (Table 4).

Table 3 Yearly transition matrices. .

(a) From 1963 to 77 in Lambir National Park. We couldn't obtain the matrix at the first stage

	1963				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland					
Secondary Forest					
Natural Forest			NONE		
Selectively logged forest					
Plantation					

(b) From 1977 to 97 in Lambir National Park. Though the matrix includes negatibe element, it is only appropriate solution among 20^5 solutions.

	1977				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland	0.906	0.004	0.002	−0.002	−0.020
Secondary Forest	0.095	0.995	−0.019	0.032	0.180
Natural Forest	0.000	0.000	0.960	0.000	0.000
Selectively logged forest	0.000	0.000	0.045	0.970	0.000
Plantation	−0.001	0.000	0.011	0.000	0.840

(c) Deramakot Forest Reserve in Malaysia. Though the matrix includes negatibe element, it is only appropriate solution among 17^5 solutions.

	1985				
	Cropland	Secondary Forest	Natural Forest	Selectively logged forest	Water
Cropland	0.965	0.052	0.041	0.022	0.043
Secondary Forest	0.004	0.712	0.001	−0.001	−0.002
Natural Forest	0.006	0.251	0.915	0.021	0.004
Selectively logged forest	0.025	−0.026	0.045	0.958	−0.001
Water	0.001	0.012	0.000	0.000	0.955

Table 4 Ten-year transition matrices in three research sites.

(a) From 1963 nto 77 in Lambir National Park. We couldn't obtain the matrix at the first stage,

	1963				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland					
Secondary Forest					
Natural Forest			NONE		
Selectively logged forest					
Plantation					

(b) From 1977 nto 97 in Lambir National Park. The elements in matrices are sometimes negative.

	1977				
	Cropland	Secondary Forest	Natural forest	Selectively logged forest	Plantation
Cropland	0.383	0.027	0.005	−0.005	−0.042
Secondary Forest	0.620	0.969	−0.042	0.268	0.868
Natural Forest	0.000	0.001	0.666	0.000	−0.001
Selectively logged forest	0.000	0.001	0.324	0.738	−0.001
Plantation	−0.002	0.002	0.046	−0.001	0.176

(c) Deramakot Forest Reserve in Malaysia. The elements in matrices are sometimes negative.

	1985				
	Cropland	Secondary Forest	Natural Forest	Selectively logged forest	Water
Cropland	0.635	0.029	−0.001	0.000	0.005
Secondary Forest	−0.001	0.037	0.003	−0.001	0.010
Natural Forest	0.027	0.482	0.445	0.120	0.064
Selectively logged forest	0.033	0.141	0.284	0.698	0.189
Water	0.306	0.311	0.269	0.182	0.733

Discussion

In examining the land use change in terms of satellite images or aerial photographs, we often don't have enough number of photographs and cannot construct transition matrices with constant census intervals. We here developed a way to calculate the yearly transition matrix from a transition matrix with long census period. This way would be useful when we want to compare among transition matrices with different census intervals, as shown in Table 3 and 4. While we applied the formula for yearly transition matrix to three transition matrices in two research sites (Table 2), we are confronted with two difficulties. One is that no positive matrix is obtained in all the cases. From our experience, small negative elements in yearly transition matrices are likely to occur when many zero or small elements are included in the original matrix. For example, in Deramakot, there are 9 elements under 0.01 among 5x5 elements (Table 2c). Since the transition among classification is usually slow in forest ecosystems, there might be many small elements in the original matrix. Then, we would obtain a yearly transition matrix with negative elements close to zero, and those negative elements could be assumed to be approximately zero. It would be derived from rounding errors in numerical calculation.

In the yearly transition matrix of Lambir National Park, there were large negative values in Table 3e, i.e. the transition probability from “plantation” to “cropland” and that from “natural forest” to “secondary forest”. It’s difficult to think those negative values are derived from rounding errors in numerical calculation. One of the possibilities is that GIS software made a mistake in the classification of land use and improbable transition was picked up from photographs. It is also probable that the drastic change of land use occurred during the census period and it is not adequate to calculate the average transition rate (yearly transition matrix). We have not identified the causes yet.

The other difficulty is that we couldn’t obtain the yearly transition matrix at the first stage in Lambir National Park (Table 3a), Mathematically speaking, the reason is why two of the eigenvalues of the original transition matrix are zero and equation (2) could not be applied to the matrix. It also means that ”selectively logged forest” and “Plantation” of the land use classification newly appeared in 1977. The appearance of new land-use classification could occur occasionally where human activity is strong. Therefore, a question remains unsolved, how to obtain yearly transition matrix where there are zero eigenvalues of the original matrix.

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