When Is It Optimal to Exhaust a Resource in a Finite Time?*

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Introduction

Although it may sound paradoxical, in order to realize sustainable resource use, we have to consider the rationale for *un*sustainable resource use. It is important to understand under what conditions a *rational* agent chooses unsustainable resource use, because an effective policy for sustainability should be one that removes the very conditions that render unsustainable use rational. This study identifies the conditions under which finite-time exhaustion of a renewable resource is optimal.

As well known, there is a great deal of debate on how to define sustainability, sustainable development, and sustainable resource management (See, for example, Pearce et al., 1990). However, finite-time exhaustion obviously defies sustainability. As the source of finite-time resource extinction, we discuss the following aspects: (a) the discounting of future benefits, (b) uncertainty about the future of the resource stock, (c) nonconvexity of natural growth function, (d) socio-psychological aspect of work incentives, and (e) strategic interaction among resource users. The policy implications are also discussed.

Intrinsic growth rate and uncertainty

Consider a bio-economic model:

$$\max_{c(t)\geq 0} \int_0^\infty u(c(t)) e^{-\rho t} dt \tag{1.a}$$

$$\text{subject to } \dot{x}(t) = f(x(t)) - c(t), \tag{1.b}$$

$$x(t), c(t) \geq 0,\ x(0) = x_0 \text{ given}, \tag{1.c}$$

where x denotes the stock of a renewable resource. The natural growth of the resource is described by function $f(x)$. Variable $c(t)$ denotes the amount of harvest at time t. $\dot{x}(t)$ denotes the time derivative of $x(t)$. The consumption of harvest yields utility to the resource user according to the utility function $u(c)$. The *effective* discount rate ρ is the sum of the user's time preference rate and the hazard rate for a fatal event such as complete destruction of the ecosystem or confiscation of the property right.

Under the assumption that the natural growth function is concave, if the intrinsic growth rate $f'(0)$ is too low or if the effective discount rate ρ is too high, then resource extinction is an optimal policy. It is worth noting that instability of the social system and/or ecosystem increases the hazard rate of a fatal event and thus the effective discount rate ρ. Therefore, for sustainable resource use, it is important to achieve institutional, political and economic stability as well as to maintain ecological resistance and resilience.

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Convex-concave growth function and the initial stock level of the resource

Assume that the natural growth function exhibits a convex-concave shape due to, for example, the Allee effect. (See Figure 1.) Also, assume that the marginal growth rate at origin is less than the effective discount rate. Given these assumptions, there is a threshold of the stock level x_C such that if the initial stock level is smaller than the threshold, then resource extinction is an optimal policy. (See Figure 2.) This implies that if resource degradation is overlooked, eventually sustainable resource management will no longer accord with social welfare maximization.

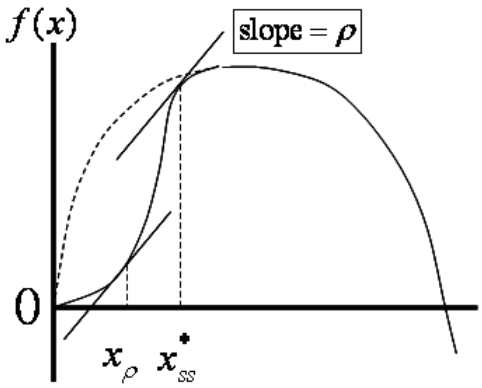

Figure 1 Convex-concave natural growth function.

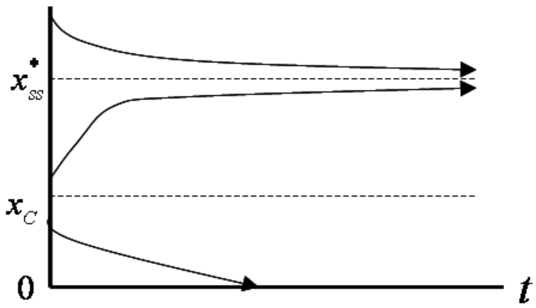

Figure 2 Optimal paths when the natural growth function is convex-concave.

Non-pecuniary value of employment

We modify the utility function in (1.a) to take into account that the people gain their utility not only from consumption but also from working. Formally, we assume:

$$u(c,E),\ E \in [0,\bar{E}],\ E = \text{ working time. } \partial u / \partial E > 0, \partial u^2 / \partial E^2 < 0.$$

If the resource extraction sector is the only industry in the economy and the harvest technology is a concave function in efforts ($E = E(c),\ E' > 0,\ E'' > 0$), then resource extinction with the maximal harvesting effort is optimal, independent of the magnitudes of discount rate and the initial stock level of resource. We will refer to this situation as *full-employment obsession*. For detailed analysis and discussion, see Farzin and Akao (2006).

Ill-defined property right

The “tragedy of the commons,” a problem of a common property resource used by multiple resource users, emerges when the property right of the resource is ill-defined and the resource is freely used by the resource users. The simplest formulation of the problem is as follows:

$$\max_{c(t)\geq 0} \int_0^{\infty} u(c(t))e^{-\rho t}dt \tag{2.a}$$

$$\text{subject to } \dot{x}(t) = f(x(t)) - (n-1)\sigma(x) - c(t), \tag{2.b}$$

$$c(t) \in [0, \bar{h}], \; x(0) = x_0 \text{ given}, \tag{2.c}$$

where $\sigma(x)$ is the common harvesting strategy of all other users, which is assumed to depend only on the resource stock, and in particular, to be time independent. Therefore, in this simplest case, we assume that every user adopts the same stationary Markovian strategy. If the solution of the above problem is given by the same strategy as other users’ strategy $\sigma(x)$, then no player has an incentive to change the strategy. A profile of such a strategy is called a (symmetric) Nash equilibrium. A Nash equilibrium strategy describes the resource use realized by rational resource users.

Under the condition $f(x) < n\bar{h}$ for all $x \geq 0$, the strategy with the users’ maximal harvesting efforts:

$$\sigma(x) = \begin{cases} \bar{h} & \text{if} \quad x > 0 \\ 0 & \phantom{\text{if}} \quad x = 0 \end{cases}, \tag{3}$$

leads the resource to extinction at the most rapid speed and is called the *most rapid extinction strategy*. The most rapid extinction strategy constitutes a Nash equilibrium if the following inequality holds:

$$u'(\bar{h}) \geq \frac{u(\bar{h})}{n\bar{h} - f(x_{ss}^*)} \exp\left[-\rho \int_0^{x_{ss}^*} \frac{dy}{n\bar{h} - f(y)}\right]. \tag{4}$$

This result, which is proved by Sorger (1998), implies that the resource is exhausted at the most rapid speed if any of the following conditions is satisfied: (a) the time discount rate ρ is high; (b) the users are greedy in the sense that the marginal utility $u'(\bar{h})$ is high; (c) the aggregate harvest ability $n\bar{h}$ is high. This is a theoretical representation of Hardin’s (1964) “tragedy of the commons.”

Even in this simplest case, it is known that there are multiple Nash equilibria. Not only the most rapid finite time resource extinction, but also sustainable resource use can be a Nash equilibrium under condition (4). Therefore, the tragedy of the commons may not be inevitable for a common property resource. However, it is hard to predict which equilibrium arises. This indicates that for two communities with identical resource stock and individual preferences, one may use its natural resource in a sustainable way, whereas the other may exhaust its resource at the most rapid speed. It is also possible that a community that has been using its resource in a sustainable way for a period of time may suddenly switch to a ruinous resource use path without any evident trigger.

Policies to Avoid Unsustainable Resource Use

The risk of a fatal event for resource management raises the effective discount rate and a high discount rate brings finite-time resource extinction. To prevent such a situation, we need to mitigate the risk caused by socio or ecological instability. For the socio-economic dimension, political stability matters. For a fragile ecosystem, unsustainable resource use is more likely to become a rational choice, and thus careful attention is required.

If the growth function of the resource exhibits nonconvexity and the resource is being degraded, a policy to mitigate the risk of a fatal event should be implemented early on, because when the resource has been already degraded, extinction is more prone to be an optimal resource use policy, even with a low discount rate.

Non-pecuniary value of employment may make people give priority to full employment over sustainable resource use. Farzin and Akao (2006) show that the remedy is none but to create alternative employment sources to absorb labor force which is excessive from the viewpoint of sustainable resource use. They also suggest that earlier policy implementation is more prudent, since when the resource is more degraded, higher wage rates may be necessary to prevent resource exhaustion.

A direct solution to the problems stemming from an ill-defined property right of resource is privatization. However, this solution may be physically difficult for some resources such as the global atmosphere and migratory animals. Even if privatization is possible, Dasgupta and Mäler (1997) give a caution that it may bring further resource degradation. This is due to the existing inequality in a rural community. If the resource is not favorably distributed to the poor, they cannot help but to encroach on the resource. As for other policy measures, Akao (2007) shows that among standard economic policy measures, a tax on harvest fails to avoid an unsustainable Nash equilibrium, whereas a system of tradable permits or quotas works well to realize sustainable and efficient resource usage.

Finally, resource-sector technological assistance and income assistance may not help to prevent finite-time extinction. In particular, if technological assistance improves harvesting efficiency, and hence the maximum harvesting ability, it may even accelerate resource extinction.

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